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Threshold Corrections to the Minimal SUSY SU(5) Grand Unified Theory

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Threshold corrections have been carried out at low- and high-energy scales to the minimal supersymmetric SU(5) grand unified theory. More refined values for the Weinberg angle and strong coupling constant are predicted which are fully consistent with the latest experimental values.

1. INTRODUCTION

The minimal supersymmetric (Wess and Zumino, 1974) SU(5) grand unified theory (Georgi and Glashow, 1974; Georgi *et al.*, 1974; Buras *et al.*, 1978; Dimpoulos and Georgi, 1981; Sakai, 1981; Witten, 1981) was successful in unifying (Pati and Salam, 1973a,b, 1974) strong, weak, and electromagnetic interactions. It could predict the Weinberg angle (Langacker and Mann, 1989) up to two decimals correctly even at the one-loop level and the strong coupling constant up to two decimals correctly at the two-loop level consistent with the experimental values at the M_Z scale (Ellis *et al.*, 1990, 1991; Amaldi *et al.*, 1991; Anselmo *et al.*, 1991; Langacker and Luo, 1991). In this communication we derive the Weinberg angle up to four decimals (Review of Particle Properties, 1994) correctly, which is 0.2319, and the strong coupling constant up to three decimals (Review of Particle Properties, 1994) correctly, which is 0.120, with full threshold corrections (Weinberg, 1980; Hall, 1981) at low- and high-energy scales to the minimal SUSY SU(5) GUT model (Langacker and Polonsky, 1993; Hagiwara and Yamada, 1993). Here, the top quark is assumed to be at 176 GeV/c^2 as claimed by the CDF group of Fermilab in 1995 (Abe *et al.*, 1994a,b, 1995; Abachi *et al.*, 1995). In our calculation,

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Table I. $SU(3)_c \times SU(2)_L \times U(1)_Y$ Quantum Numbers, Masses, and β-Function Coefficients $b_i(j)$ of the Heavy Multiplets j in the Minimal Supsersymmetric SU(5) Model Gauge Sector

j	R	Mass	$b_1(j)$	$b_2(j)$	$b_3(j)$
$\underline{X}$, $\underline{Y}$	(3, 2, ± 5/6)	m_x	−35/4	−21/4	−7/2
X, Y	(3, 2, ± 5/6)	m_x	10/3	2	4/3
H_X, H_Y	(3, 2, ± 5/6)	m_x	5/12	1/4	1/6
Sum		m_x	−5	−3	−2

superheavy masses at the grand unification scale (Weinberg, 1980), supersymmetric effective masses closer to the M_Z scale (Langacker and Polonsky, 1993), the heavy top quark (Degrassi *et al.*, 1991; Hall, 1981; Sirlin, 1994), top Yukawa coupling (Langacker and Polonsky, 1993), and quantum gravitational effects through five-dimensional nonrenormalizable operators (Hill, 1984; Shafi and Wetterich, 1984) with unit coefficients play a crucial role.

The heavy sector of the minimal SUSY SU(5) GUT model (Masiero *et al.*, 1982; Hagiwara and Yamada, 1993) has the superpotential $W = m_{24}\mathrm{Tr}(\Sigma^2) + \lambda_1\mathrm{Tr}(\Sigma^3) + \lambda_2\bar{\Phi}\Sigma\Phi + m_5\bar{\Phi}\Phi$ with three chiral supermultiplets: $\Sigma(24)$, $\Phi(5)$, and $\bar{\Phi}(\bar{5})$. By choosing the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetric vacuum, $\langle\Sigma\rangle = V_{24}\mathrm{diag}(-2, -2, -2, 3, 3)/2\sqrt{15}$ with $V_{24} = -4\sqrt{15}m_{24}/3\lambda_1$ and $\langle\Phi\rangle = \langle\bar{\Phi}\rangle = 0$, we find the mass spectrum (Hagiwara and Yamada, 1993) of Tables I and II after making the fine tuning $2(\lambda_2/\lambda_1)m_{24} - m_5 = 0$.

The (3, 2, ±5/6) components of Σ combine with the corresponding (X, Y) components of the gauge multiplet to make the gauge-Higgs supermultiplet of Table I with the common mass $m_x^2 = (5/6)g_5^2V_{24}^2$. The rest of Σ has either the mass $5m_{24}(= m_\Sigma)$ or m_{24} as listed in Table II. The triplet components of Φ and $\bar{\Phi}$ shown as D in Table II have a common mass $m_D = (5/3)m_5$. Under the fine tuning condition, the superpotential has three free parameters which can be parametrized by the three physical masses: the mass of the (X, Y) gauge-Higgs supermultiplet m_X, the largest mass m_Σ of the remaining components of Σ, and the triplet mass m_D.

Table II. Higgs Sector of the Minimal Model with 24, 5, and $\bar{5}$

j	R	Mass	$b_1(j)$	$b_2(j)$	$b_3(j)$	Comments
$H_{(8,1)}$, $\bar{H}_{(8,1)}$	(8, 1, 0)	m_Σ	0	0	3/2	in 24
$H_{(1,3)}$, $\bar{H}_{(1,3)}$	(1, 3, 0)	m_Σ	0	1	0	
$H_{(1,1)}$, $\bar{H}_{(1,1)}$	(1, 1, 0)	$0.2m_\Sigma$	0	0	0	
D, $\bar{D}$	(3, 1, ±1/3)	m_D	1/5	0	1/2	in 5, $\bar{5}$

Now consider the two-loop renormalization group equation

$$\mu\, \partial\alpha_i/\partial\mu = (b_i/2\pi)\alpha_i^2 + \sum_{j=1}^{3} (b_{ij}/8\pi^2)\alpha_i^2\alpha_j \qquad \text{for} \quad i = 1, 2, 3$$

where α_i for $i = 1, 2, 3$ are the normalized coupling constants of the electromagnetic, weak, and strong interactions, respectively, and μ is the scale parameter. Further, b_i and b_{ij} are the one-loop and two-loop β-function coefficients (Jones, 1982), with values given by

$$b_i = \begin{pmatrix} 33/5 & & \\ & 1 & \\ & & -3 \end{pmatrix}, \qquad b_{ij} = \begin{pmatrix} 7.96 & 5.4 & 17.6 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{pmatrix}$$

The solution to the above equation is given by

$$1/\alpha_i(M_Z) = 1/\alpha_G + b_i t + \theta_i - \Delta_i \qquad \text{for} \quad i = 1, 2, 3$$

where $t = (1/2\pi)\ln(M_G/M_Z)$, $\theta_i = 1/4\pi\ \Sigma_{j=1}^{3}\ (b_{ij}/b_j)\ln[\alpha_j(M_G)/\alpha_j(M_Z)]$, Δ_i is the threshold and other corrections, M_G is the grand unified mass, M_Z is the mass of the Z-vector boson, and α_G is the unified coupling constant. Δ_i should be calculated to a precision consistent with the θ_i.

At the Z threshold we have $1/\alpha_i(M_Z) = (3/5)[1 - s^2(M_Z)]/\alpha(M_Z)$, $s^2(M_Z)/\alpha(M_Z)$, and $1/\alpha_s(M_Z)$ for $i = 1, 2, 3$, respectively. $s^2(M_Z)$, $\alpha(M_Z)$, and $\alpha_s(M_Z)$ are the weak angle, electromagnetic coupling, and strong coupling constants, respectively, the three low-scale parameters which are defined in the modified minimal subtraction scheme ($\overline{\text{MS}}$) (Degrassi *et al.*, 1991) and evaluated at the Z-pole. Here M_G serves as the high-scale boundary of the desert, while M_Z serves as the low-scale boundary of the desert.

Here it is assumed that $\alpha_1(M_G) = \alpha_2(M_G) = \alpha_3(M_G) = \alpha_G$, the unified coupling constant.

The two-loop terms can be rewritten using the lowest order solution for the couplings, i.e.,

$$1/\alpha_i(M_Z) = 1/\alpha_G + b_i t; \qquad \alpha_G(\equiv \alpha_i(M_G))/\alpha_i(M_Z) = (1 + b_i t\alpha_G)$$

$\theta_i = 1/4\pi\ \Sigma_{i=1}^{3}\ (b_{ij}/b_j)\ln(1 + b_j\alpha_G t)$ for $i = 1, 2, 3$, where the one-loop expressions for α_G and t are to be substituted.

Using the expression $\theta_i = 1/4\pi\ \Sigma_{j=1}^{3}\ (b_{ij}/b_j)\ln(1 + b_j\alpha_G t + \theta_j\alpha_G)$ for $i = 1, 2, 3$, we can calculate the two-loop terms up to two-loop level.

The correction terms Δ_i for $i = 1, 2, 3$ are given by

$$\Delta_i = \Delta_i^{\text{conversion}} + \sum_{\text{boundary}} \sum_{\zeta} (b_i^{\zeta}/2\pi)[\ln(M_\zeta/M_{\text{boundary}}) - C^{J_\zeta}]$$
$$+ \Delta_i^{\text{top}} + \Delta_i^{\text{Yukawa}} + \Delta_i^{\text{NRO}}$$

The first term is a constant, which depends only on the gauge group G,

$$\Delta_i^{\text{conversion}} = -C_2(G_i)/12\pi$$

where $C_2(G_i)$ is the quadratic Casimir operator for the adjoint representation $C_2(G_i) = N$ for $G_i = SU(N)$ and $= 0$ for $G_i = U(1)$. The term $\Delta_i^{\text{conversion}}$ results from the need to use the dimensional-reduction (DR) scheme in the MSSM, so that the algebra is kept in four dimensions. Thus we convert the $\overline{\text{MS}}$ coupling above M_Z, $1/\alpha_i^{\overline{\text{MS}}} = 1/\alpha_i^{\overline{\text{DR}}} - \Delta_i^{\text{conversion}}$.

The second term in the expression for Δ_i sums over the one-loop threshold corrections, b_i^ζ is the (decoupled) contribution of a heavy field to the β function coefficient b_i between M_ζ and M_{boundary}. C^{J_ζ} is a mass-independent number, which depends on the spin J_ζ of ζ and on the regularization scheme used. In $\overline{\text{MS}}$ (using dimensional regularization) one has $C^1_{\overline{\text{MS}}} = 1/21$, $C^{1/2}_{\overline{\text{MS}}} = C^0_{\overline{\text{MS}}} = 0$. These are to be used at the low-scale boundary, while at the other boundary (using dimensional reduction) we have $C^J_{\overline{\text{DR}}} \equiv 0$.

The above summation has to be done at the low-scale boundary in the minimal supersymmetric standard model (MSSM) which is embedded in the minimal SUSY SU(5) grand unified theory for these particles and heavy Higgs doublet. Instead of considering the individual masses of these particles, which can be calculated given a small number of high-scale parameters, i.e., a universal gaugino mass $m_{1/2}$, a universal scalar mass m_0, the Higgs mixing parameter μ_{mixing}, a universal trilinear coupling A, and the top Yukawa coupling h_t (here we omit all other Yukawa couplings) by solving a set of coupled renormalization-group equations (RGEs) (other mass parameters, such as the universal bilinear coupling B, are related to the parameters above by boundary conditions and the constraint setting the weak breaking scale), we use a parametrization in terms of three low-energy effective parameters defined by (Langacker and Polonsky, 1993)

$$\sum_\zeta (b_i^\zeta/2\pi) \ln(M_\zeta/M_Z) = (b_i^{\text{MSSM}} - b_i^{\text{SM}})/2\pi \ln(M_i/M_Z) \qquad \text{for} \quad i = 1, 2, 3$$

where

$$b_i^{\text{SM}} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}, \qquad b_i^{\text{MSSM}} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix}$$

In this paper we use the following data (Review of Particle Properties, 1994) from principal LEP and other recent observations:

$$M_Z = 91.187 \pm 0.007 \text{ GeV}, \qquad 60 \text{ GeV} < M_H < 1 \text{ TeV}$$

the global best fit values

$$m_t = 169^{+16+17}_{-18-20} \text{ GeV}$$

$$\hat{S}_Z^2 = 0.2319 \pm 0.0005 \pm 0.0002$$

$$\alpha_S(M_Z) = 0.120 \pm 0.007 \pm 0.002$$

where the central values are for a Higgs mass of 300 GeV, and the second error bars for $m_H \to 1000\ (+)$ or $60\ (-)$.

In the modified minimum subtraction ($\overline{\text{MS}}$) scheme (Degrassi *et al.*, 1991; Sirlin, 1994) $\hat{\alpha}(M_Z)^{-1} = 127.9 \pm 0.1$.

We will now discuss the threshold corrections (Hall, 1981) due to the heavy top quark. In the $\overline{\text{MS}}$ scheme to account for $m_t > M_Z$ one can define threshold corrections to $\alpha(M_Z)$ and $\alpha_s(M_Z)$, i.e., $(b_Q^{\text{top}}/2\pi)\ln(m_t/M_Z)$ and $(b_3^{\text{top}}/2\pi)\ln(m_t/M_Z)$, respectively where b_Q^{top} and b_3^{top} are the top contributions to the relevant one-loop β-function slope. In $\overline{\text{MS}}$ the definition for our central value $m_t = 169$ GeV, our value of $\alpha(M_Z)$ already includes the top threshold correction, and we have to further correct $\alpha(M_Z)$ only for different values of m_t.

Thus

$$\Delta_\alpha^{\text{top}} = (8/9\pi)\ln(m_t/169 \text{ GeV})$$

$$\Delta_{\alpha_s}^{\text{top}} = (1/3\pi)\ln(m_t/91.187 \text{ GeV})$$

Similarly, the m_t threshold corrections are already included in the $S^2(M_Z)$ definition. However, the input value of $S_0^2(M_Z)$ extracted from the data depends both quadratically and logarithmically on m_t. In particular the value $S_0^2(M_Z) = 0.2319 \pm 0.0005$ is for the best fit value $m_t = m_{t0} = 169$ GeV. For other m_t the corresponding $S^2(M_Z)$ is (Degrassi *et al.*, 1991)

$$S^2 = S_0^2 - (3G_F/8\sqrt{2}\pi^2)S_0^2[(1 - S_0^2)/(1 - S_0^2)](m_t^2 - m_{t0}^2)$$

where $G = 1.166392 \times 10^{-5}$ GeV^{-2} is the Fermi coupling, and we have neglected logarithmic dependences on m_t.

We then have

$$S^2(M_Z) = S_0^2(M_Z) + \Delta_{S^2}^{\text{top}},$$

where

$$\Delta_{S^2}^{\text{top}} \equiv -1.041 \times 10^{-7}\,[m_t^2 - (169)^2]$$

$$S^2(m_Z) = 0.2319 - 1.041 \times 10^{-7}\,(m_t^2 - 28{,}561)$$

The m_t dependence of the “true” $S^2(M_Z)$ is $\Delta_{S^2}^{\text{top}}$ and will be included together with the threshold corrections in Δ_i^{top}. Thus

$$\Delta_1^{\text{top}} = \{8[1 - S_0^2(M_Z)]/15\pi\} \ln(m_t/169 \text{ GeV}) - (3/5)\Delta^{\text{top}}_{S^2/\alpha(M_Z)}$$

$$\Delta_2^{\text{top}} = [8S_0^2(M_Z)/9\pi] \ln(m_t/169 \text{ GeV}) + \Delta^{\text{top}}_{S^2/\alpha(M_Z)}$$

$$\Delta_3^{\text{top}} = (1/3\pi) \ln(m_t/91.187 \text{ GeV})$$

Another issue that is related to the heavy top is the contribution of the top Yukawa coupling h_t to the two-loop β function. If $h_t = 1$, we have to reintroduce the relevant term (that was neglected above) in the β function, i.e.,

$$\mu\, \partial\alpha_i/\partial\mu = (b_i/2\pi)\alpha_i^2 + \sum_{j=1}^{3} (b_{ij}/8\pi^2)\alpha_i^2\alpha_j - b_{i,\text{top}}(h_t^2/16\pi)\alpha_i^2/2\pi$$

where $b_{i,\text{top}} = 26/5,\ 6,\ 4$ for $i = 1, 2, 3$ in the minimal supersymmetric standard model. h_t is running and is coupled to α_i at the one-loop order, and the Δ_i^{Yukawa} are functions of the coupling h_t and α_G at the unification point, and of the unification point parameter t, and have to be calculated numerically.

Here we use an approximation in which h_t is constant. Then the new term in the above equation is realized as a negative correction to b_i, and

$$\Delta_i^{\text{Yukawa}} = b_{i,\text{top}}(h_t^2/16\pi^2)t \qquad \text{for} \quad i = 1, 2, 3$$

$h_t = 1 = h_{\text{fixed}}$ is a reasonable approximation (h_{fixed} is the fixed point of the one-loop top Yukawa renormalization-group equation).

Finally we consider contributions from nonrenormalizable operators at the high scale which may be induced by the physics between M_G and $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV/C^2. We consider only dimension-five operators;

$$-(1/2)(\eta/M_{\text{Planck}})\ \text{Tr}(F_{\mu\nu}\Sigma F^{\mu\nu})$$

where η is a dimensionless parameter and $F_{\mu\nu}$ is the field strength tensor. In the minimal SUSY SU(5) model, Σ is the 24-real Higgs (Majorana super) multiplet (contributions from higher dimension operators are suppressed by power of M_{Planck}^{-1}). When Σ acquires an expectation value the effect is to renormalize the gauge fields, which can be absorbed into a redefinition of the couplings. The running couplings at M_G are related to the underlying gauge coupling $\alpha_G(M_G)$ by (Hill, 1984; Shafi, 1984) $1/\alpha_i(M_G) = (1 + \varepsilon_i)/\alpha_G$, where $\varepsilon_i = \eta k_i \sqrt{r/\pi\alpha_G}(M_G/M_{\text{Planck}})$.

In this model $r = 2/25$ and $k_i = 1/2,\ 3/2,\ -1$ for $i = 1, 2, 3$, respectively. We treat these operators perturbatively (i.e., for $|\eta| < 10$), by defining $\Delta_i^{\text{NRO}} = -\eta k_i[r/(\pi\alpha_G^3)]^{1/2}(M_G/M_{\text{Planck}})$, where it is sufficient to use the one-loop expressions for α_G and $M_G = M_Z e^{2\pi t}$. Therefore

$$\Delta_i^{\text{NRO}} = -\eta k_i[2/(25\pi\alpha_G^3)^{1/2}](0.74743443) \times 10^{-17} e^{2\pi t}$$

Substituting values for b_i^{MSSM} for $i = 1, 2, 3$, we obtain the following two sets of equations as predictions for t, α_G^{-1}, S^2 (m_Z) and t, α_G^{-1}, $\alpha_S(M_Z)$, respectively:

(A′)

$$t = (1/60)[3/\alpha(M_Z) - 8/\alpha_S(M_Z)] - (1/60)(-5\theta_1 + 3\theta_2 - 8\theta_3)$$
$$+ (1/60)(5\Delta_1 + 3\Delta_2 - 8\Delta_3)$$
$$1/\alpha_G = (3/20)[1/\alpha(M_Z) + 4/\alpha_S(M_Z)\ ] - (1/20)(12\theta_3 + 5\theta_1 + 3\theta_2)$$
$$+ (1/20)(12\Delta_3 + 5\Delta_1 + 3\Delta_2)$$
$$S^2(M_Z) = 0.2 + (7/15)\alpha(M_Z)/\alpha_S\ (M_Z) + [\alpha(M_Z)/60](48\theta_2 - 28\theta_3 - 20\theta_1)$$
$$+ [\alpha(M_Z)/60](20\Delta_1 - 48\Delta_2 + 28\Delta_3)$$

(B′)

$$t = [3 - 8S_0^2\ M_Z)/(28\alpha(M_Z)] + (5/28)(\theta_2 - \theta_1) + (5\ /28)(\Delta_1 - \Delta_2)$$
$$1/\alpha_G = [3 - 36S_0^2(M_Z)]/[28\alpha(M_Z)] + (5\theta_1 - 33\theta_2)/28 + (33\Delta_2 - 5\Delta_1)/28$$
$$\alpha_S(M_Z) = 7\alpha(M_Z)/[15S_0^2\,(M_Z) - 3] + \{28[\alpha(M_Z)]^2/[60S_0^2(M_Z) - 12]^2\}$$
$$\times (48\theta_2 - 20\theta_1 - 28\theta_3) + \{28[\alpha(M_Z)]^2/(60S_0^2(M_Z) - 12)^2\}$$
$$\times (28\Delta_3 + 20\Delta_1 - 48\Delta_2)$$

where

$$\Delta_1 = 0.0325201 - (5/\pi)\ln(m_x/M_G) + (1/5\pi)\ln(m_5/M_G)$$
$$+ (5/4\pi)\ln(M_1/M_Z)$$
$$\Delta_2 = 0.5123 - (3/\pi)\ln(m_x/M_G) + (1/\pi)\ln(m_{24}/M_G)$$
$$+ (25/12\pi)\ln(M_2/M_Z)$$
$$\Delta_3 = 0.8497504 - (2/\pi)\ln(m_x/M_G) + (3/2\pi)\ln(m_{24}/M_G)$$
$$+ (1/2\pi)\ln(m_5/M_G) + (2/\pi)\ln(M_3M_Z)$$
$$(\Delta_1 - \Delta_2) = -0.4797799 - (3/\pi)\ln(m_x/M_G)$$
$$+ (1/5\pi)\ln(m_5/M_G) - (1/\pi)\ln(m_{24}/M_G)$$
$$+ (5/4\pi)\ln(M_1/M_Z) - (25/12\pi)\ln(M_2/M_Z)$$
$$(33\Delta_2 - 5\Delta_1) = 16.7433 - (74/\pi)\ln(m_x/M_G)$$
$$+ (33/\pi)\ln(m_{24}/M_G) - (1/\pi)\ln(m_5/M_G)$$
$$+ (275/4\pi)\ln(M_2/M_Z) - (25/4\pi)\ln(M_1/M_Z)$$

Table III

Physical quantity	Values up to one-loop level using set (A′)	Values up to one-loop level using set (B′)
t	5.2838889	5.2292829
M_G	2.3897875×10^{16} GeV/c^2	1.6957156×10^{16} GeV/c^2
α_G^{-1}	24.185	24.430727
α_G	0.0413479	0.040932
$S^2(M_Z)$	0.2304056	0.2319 (input value)
$\alpha_S(m_Z)$	0.120 (input value)	0.1143783

$$\begin{aligned}
(5\Delta_1 + 3\Delta_2 - 8\Delta_3) = {} & 8.4975037 - (18/\pi)\ln(m_x/M_G) \\
& - (3/\pi)\ln(m_5/M_G) - (9/\pi)\ln(m_{24}/M_G) \\
& + (25/4\pi)\ln(M_1/M_Z) + (25/4\pi)\ln(M_2/M_Z) \\
& - (16/\pi)\ln(M_1/M_Z) \\
(5\Delta_1 + 3\Delta_2 + 12\Delta_3) = {} & 11.896506 - (58/\pi)\ln(m_x/M_G) \\
& + (7/\pi)\ln(m_5/M_G) + (21/\pi)\ln(m_{24}/M_G) \\
& + (25/4\pi)\ln(M_1/M_Z) \\
& + (25/12\pi)\ln(M_2/M_Z) + (24/\pi)\ln(M_3/M_Z) \\
(20\Delta_1 - 48\Delta_2 + 28\Delta_3) = {} & -0.146987 - (300/\pi)\ln(m_x/M_G) \\
& + (18/\pi)\ln(m_5/M_G) - (6/\pi)\ln(m_{24}/M_G) \\
& + (25/\pi)\ln(M_1/M_Z) - (100/\pi)\ln(M_2/M_Z) \\
& + (56/\pi)\ln(M_3/M_Z)
\end{aligned}$$

Now we will do the numerical calculations using the formulas given above (see Tables III–VI).

Table IV

θ_i (for i = 1, 2, 3,)	Using one-loop values derived from set (A′)	Using one-loop values derived from set (B′)
θ_1	0.6680172	0.6476158
θ_2	1.0907898	1.0591707
θ_3	0.5608773	0.5438849

Table V

Physical quantity	Two-loop values using set (A′)	Two-loop values using set (B′)
t	5.248465	5.3027748
M_G	1.9129165×10^{16} GeV/c^2	2.6908789×10^{16} GeV/c^2
α_G^{-1}	23.517851	23.298064
α_G	0.0425208	0.042922
$S^2(M_Z)$	0.2334409	0.2319 (input value)
$\alpha_S(M_Z)$	0.120 (input value)	0.1249658

We define $\delta_i = \Delta_i^{\text{conversion}} + \Delta_i^{\text{top}} + \Delta_i^{\text{Yukawa}} + \Delta_i^{\text{NRO}}$ for $i = 1, 2, 3$.

Using the expressions given earlier for each term in this expression and using CDF data of Fermilab 1995 for the top quark mass, which is $m_t = 176 \pm 8 \pm 10$ GeV/c^2, we end up with Table VII for arbitrary η.

In this model $m_x^2 = (5/6)g_5^2 V_{24}^2$, $V_{24} = -4\sqrt{15}m_{24}/3\lambda_1$.

Therefore $m_x^2 = (5^2 2^2/3^2)2g_5^2 m_{24}^2/\lambda_1^2$; $m_x = (10/3)g_5 m_{24}$ with $\lambda_1 = \sqrt{2}$, where $g_5 = \sqrt{4\pi\alpha_G}$. Further, $m_5 = 2(\lambda_2/\lambda_1)m_{24} = \sqrt{2}\lambda_2 m_{24}$.

Therefore in the minimal model $m_x = (10/3)\sqrt{4\pi\alpha_G}m_{24}$.

Thus $m_{24} = (3/10)(1/\sqrt{4\pi\alpha_G})m_x$ and $m_5 = \lambda_2\sqrt{2}(3/10)(1/\sqrt{4\pi\alpha_G})m_x$.

Therefore $m_5 = \sqrt{2}(3/10)(1/\sqrt{4\pi\alpha_G})m_x$ with $\lambda_2 = 1$.

For $\alpha_G = 0.042$, $\sqrt{4\pi\alpha_G} \approx 0.7$ (the two-loop value).

We take $m_x = m_G$; then $m_{24} \approx 0.4m_G$, $m_5 \approx 0.5m_G$. Further, we take $M_1 = 4M_Z$, $M_2 = 4M_Z$, $M_3 = 3M_Z$. We obtain Table VIII.

Final predictions of the minimal SUSY SU(5) GUT model are made by adding the contribution in Table VIII with other contributions in Table VII (for $\eta = 1$) to the two-loop predictions in Table V, for $m_t = 176$ GeV/c^2 (see Table IX).

Finally we evaluate the grand unified energy, unified coupling constant, and proton lifetime (Seidel *et al.*, 1988; Becker-Szendy *et al.*, 1990; Hirata *et al.*, 1989; Berger *et al.*, 1991) after full threshold corrections in the minimal model for the top quark mass of 176 GeV/c^2 (see Table X).

Table VI

θ_i (for $i = 1, 2, 3$)	Using two-loop values derived from set (A′)	Using two-loop values derived from set (B′)
θ_1	0.6749068	0.6957552
θ_2	1.1551888	1.1862567
θ_3	0.580533	0.5975848

Table VII

Quantity using two-loop values under sets (A′)/(B′)	$m_t = 176$ GeV/c^2
$(1/60)(5\delta_1 + 3\delta_2 - 8\delta_3)$	(A′) 0.0058764 + (0.0004758) η
Contribution to t under A′	(B′) 0.0059452 − (0.0093951) η
$(1/20)(12\delta_3 + 5\delta_1 + 3\delta_2)$	(A′) 0.1407674 + (0.0071341) η
Contribution to α_G^{-1} under (A′)	(B′) 0.1423495 + (0.0098951) η
$(\alpha(M_Z)/60)(20\delta_1 - 48\delta_2 + 28\delta_3)$	(A′) 0.0002327 + (0.0003346) η
Contribution to $S^2(M_Z)$ under (A′)	(B′) 0.0002295 + (0.0004641) η
$(5/28)(\delta_1 - \delta_2)$	(A′) 0.0143818 + (0.0050958) η
Contribution to t under (B′)	(B′) 0.0143326 + (0.0070679) η
$(33\delta_2 - 5\delta_1)/28$	(A′) 0.1024931 − (0.0479007) η
Contribution to α_G^{-1} under (B′)	(B′) 0.1046058 − (0.0664388) η
$\{28[\alpha(M_Z)]^2/[60S^2(M_Z) - 12]^2\}(28\delta_3 + 20\delta_1 - 48\delta_2)$	(A′) 0.0008344 + (0.0011999) η
Contribution to $\alpha_s(m_Z)$ under (B′)	(B′) 0.0008229 + (0.0016642) η

2. CONCLUSIONS

With full threshold corrections to the minimal SUSY SU(5) grand unified theory at low- and high-energy scales we were successful in deriving the Weinberg angle up to four decimals correctly, which is 0.2319, and the strong coupling constant up to three decimals correctly, which is 0.120, consistent with the latest experimental values when the top quark exists at 176 GeV/c^2. The quantum gravitational effects due to 5-dimensional nonrenormalizable operators with unit coefficients played a crucial role in this derivation. Further supersymmetric effective masses and superheavy masses contributed significantly with the heavy top quark mass and top Yukawa coupling. Experimental-

Table VIII

Quantity	Value
$(1/60)(5\Delta_1 + 3\Delta_2 - 8\Delta_3)$	0.1950848
Contribution to t under (A′)	
$(1/20)(12\Delta_3 + 5\Delta_1 + 3\Delta_2)$	0.8148572
Contribution to α_G^{-1} under (A′)	
$[\alpha(M_Z)/60](20\Delta_1 - 48\Delta_2 + 28\Delta_3)$	−0.0020694
Contribution to $S^2(M_Z)$ under (A′)	
$(5/28)(\Delta_1 - \Delta_2)$	−0.1071312
Contribution to t under (B′)	
$(1/28)(33\Delta_2 - 5\Delta_1)$	1.2470879
Contribution to α_G^{-1} under (B′)	
$\{28[\alpha(M_Z)]^2/[60S^2(M_Z) - 12]^2\}(28\Delta_3 + 20\Delta_1 - 48\Delta_2)$	−0.0074199
Contribution to $\alpha_S(M_Z)$ under (B′)	

Table IX

Final predictions using two-loop values under sets (A′)/(B′)	m_t = 176 GeV/c^2
t under (A′)	(A′) 5.4499018
	(B′) 5.4939097
α_G^{-1} under (A′)	(A′) 24.48061
	(B′) 24.265166
$S^2(M_Z$ under (A′)	(A′) 0.2319383
	(B′) 0.2320651
t under (B′)	(A′) 5.1608054
	(B′) 5.2170381
α_G^{-1} under (B′)	(A′) 24.819531
	(B′) 24.583319
$\alpha_S(M_Z)$ under (B′)	(A′) 0.1195802
	(B′) 0.120033

ists can test the values derived here for proton lifetime after full threshold corrections to the minimal model.

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Table X

Final predictions using two-loop values under sets (A′)/(B′)	For m_t = 176 GeV/c^2
M_G under (A′) (grand unified mass)	(A′) 6.7821474 × 10^{16} GeV/c^2
	(B′) 8.942396 × 10^{16} GeV/c^2
α_G under (A′) (unified coupling constant)	(A′) 0.0408486
	(B′) 0.0412113
M_G under (B′) (grand unified mass)	(A′) 1.1027946 × 10^{16} GeV/c^2
	(B′) 1.570145 × 10^{16} GeV/c^2
α_G under (B′) (unified coupling constant)	(A′) 0.0402908
	(B′) 0.0406779
Proton lifetime T_P under (A′)	(A′) 2.3046603 × 10^{36} years
	(B′) 6.8434588 × 10^{36} years
Proton lifetime T_P under (B′)	(A′) 1.6559956 × 10^{33} years
	(B′) 6.6762657 × 10^{33} years

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